1.00
Lecture 27 – Random Walk
From Physics to Finance

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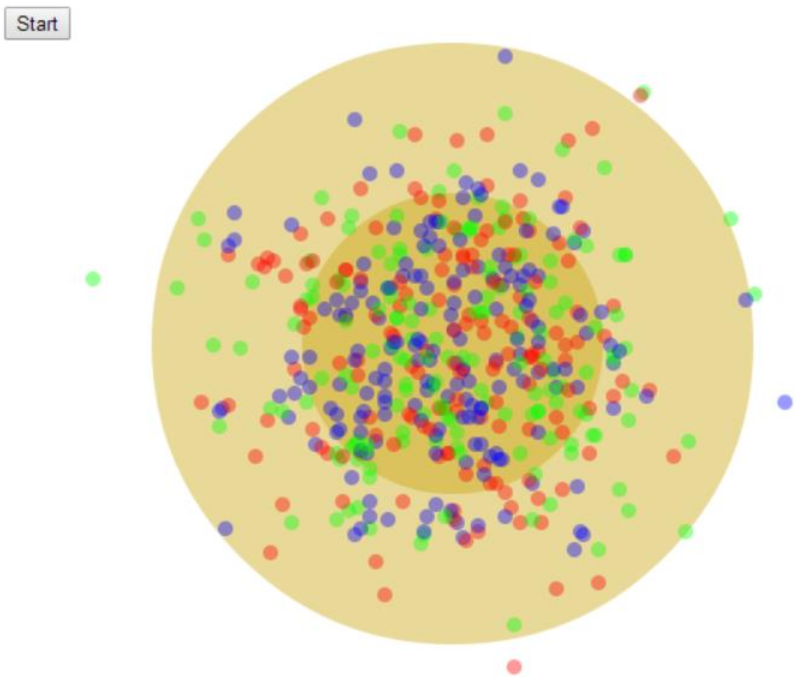
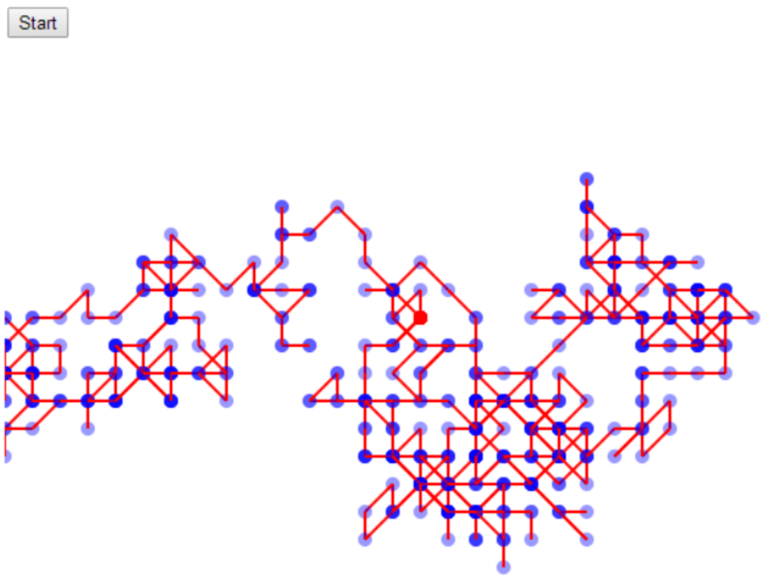
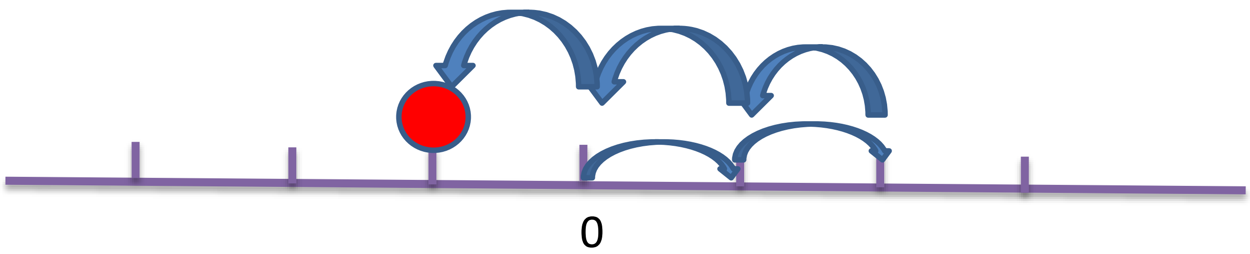




1-D Random Walk

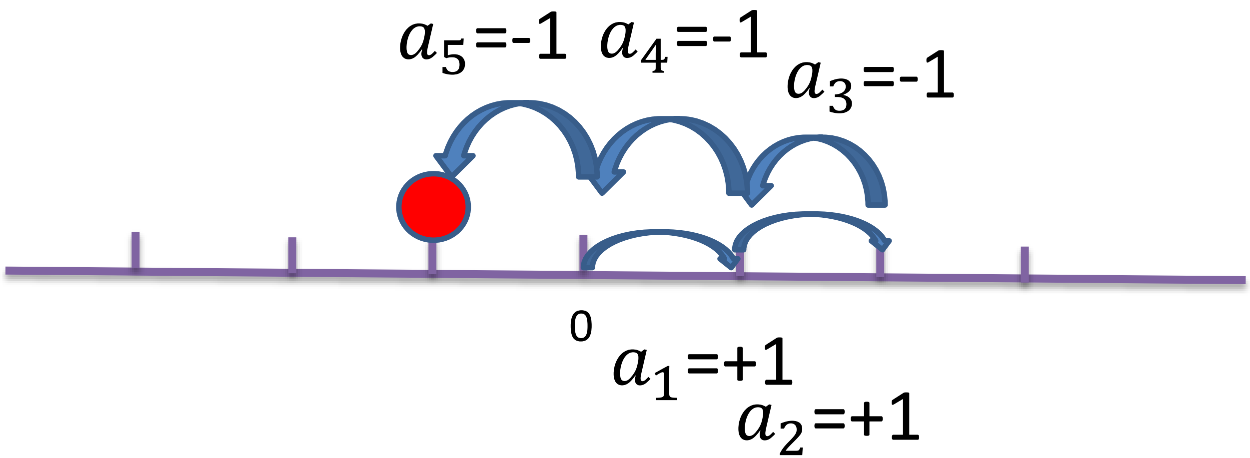
- Flip coin so that Heads moves the ball to the right and Tails moves left. Below shows H,H,T,T,T



- On average how far will it move from its starting position after N steps ?

1-D Random Walk

- Lets call the first step a_1=+1 etc. and let d be the distance moved



$$d = a_1 + a_2 + a_3 + a_4 + \ldots + a_N$$

Component 1

1-D Random Walk

- We actually want the average distance.

$$\underline{\langle d \rangle = \langle a_1 \rangle + \langle a_2 \rangle + \langle a_3 \rangle + \langle a_4 \rangle \ \ldots + \langle a_{\mathrm{N}} \rangle}$$

- The average of $\underline{\langle a_{\mathrm{N}} \rangle}$ = 0
- Thus the average is zero.

Component 1

1-D Random Walk

- Maybe we should calculate the average of $\langle d^2 \rangle$

$$\langle d^2 \rangle = \left\langle (a_1 + a_2 + a_3 + \ \ldots + a_N)^2 \right\rangle$$

$$\langle d^2 \rangle = \langle (a_1 + a_2 + a_3 + \ \ldots + a_N)(a_1 + a_2 + a_3 + \ \ldots + a_N) \rangle$$

$$\langle d^2 \rangle = (\langle {a_1}^2 \rangle + \langle {a_2}^2 \rangle + \langle {a_3}^2 \rangle .. + \langle {a_N}^2 \rangle) + 2(\langle a_1 a_2 \rangle + \langle a_1 a_3 \rangle + \langle a_1 a_4 \rangle + \langle a_2 a_3 \rangle + \ \ldots)$$

- We note that ${a_1}^2 = 1$ always
- But $\langle a_1 a_3 \rangle = 0$ on average.

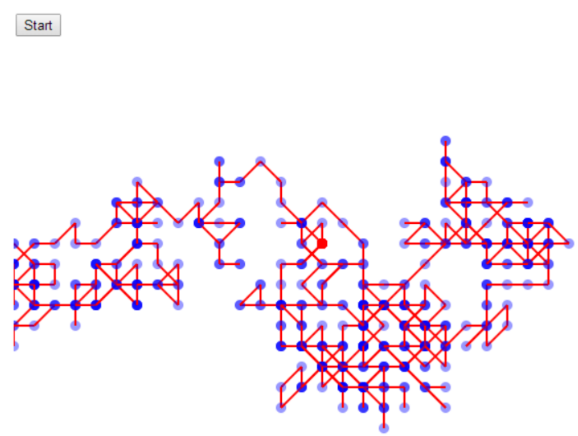
1-D Random Walk

- So $\langle d^2 \rangle = N$
- Or $\sqrt{\langle d^2 \rangle} = \sqrt{N}$

- This is called the root-mean-squared distance

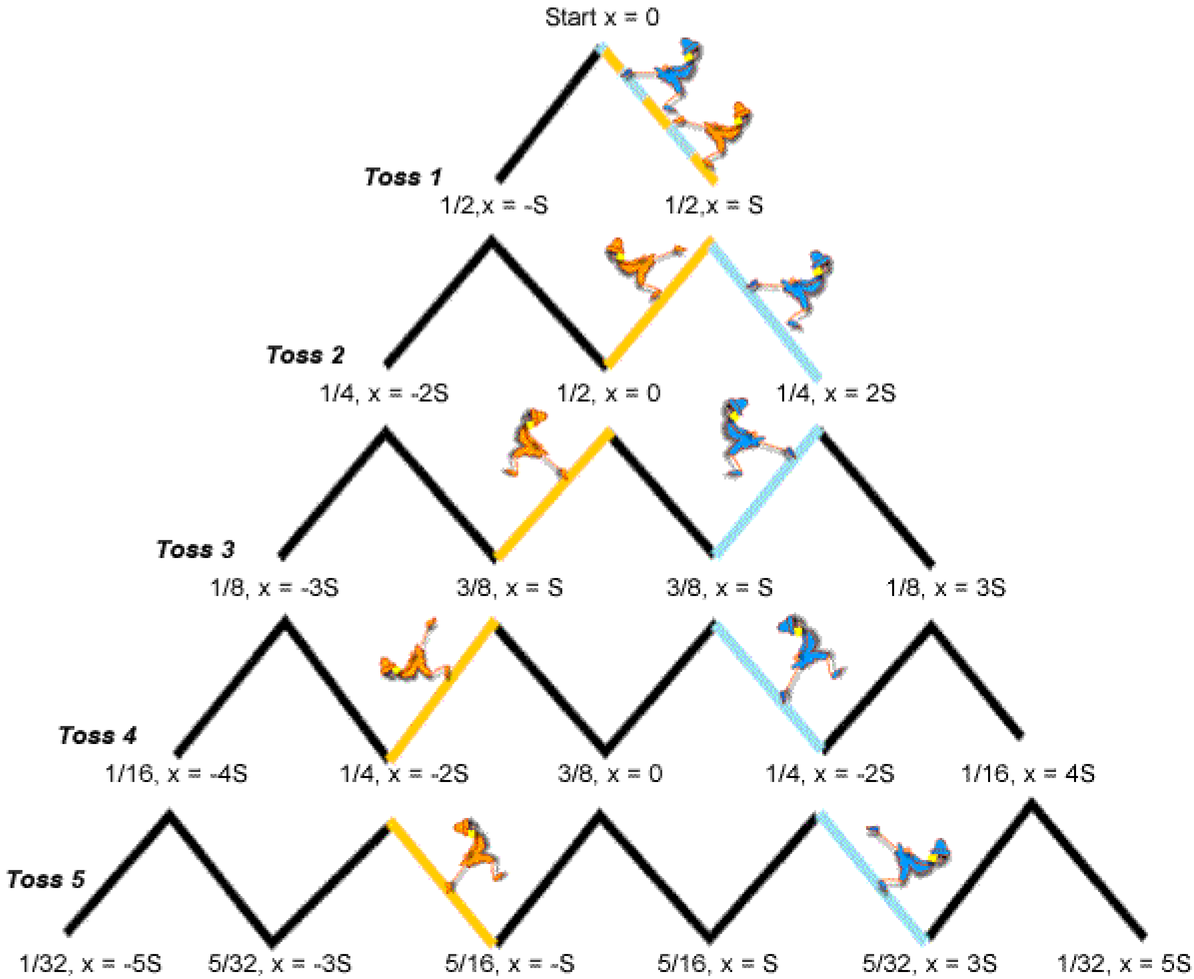
In Class Experiment

- The present code has two particles. One is fixed, the other does a random walk.
- Change the code as follows:
  - Particle moves along x-axis only
  - Each step it moves either +10 or -10 (presently it is biased)
  - Stop after 100 steps and print out distance run.
  - Repeat for 10 runs and average the distance. Hand in your answers for the 10 runs.



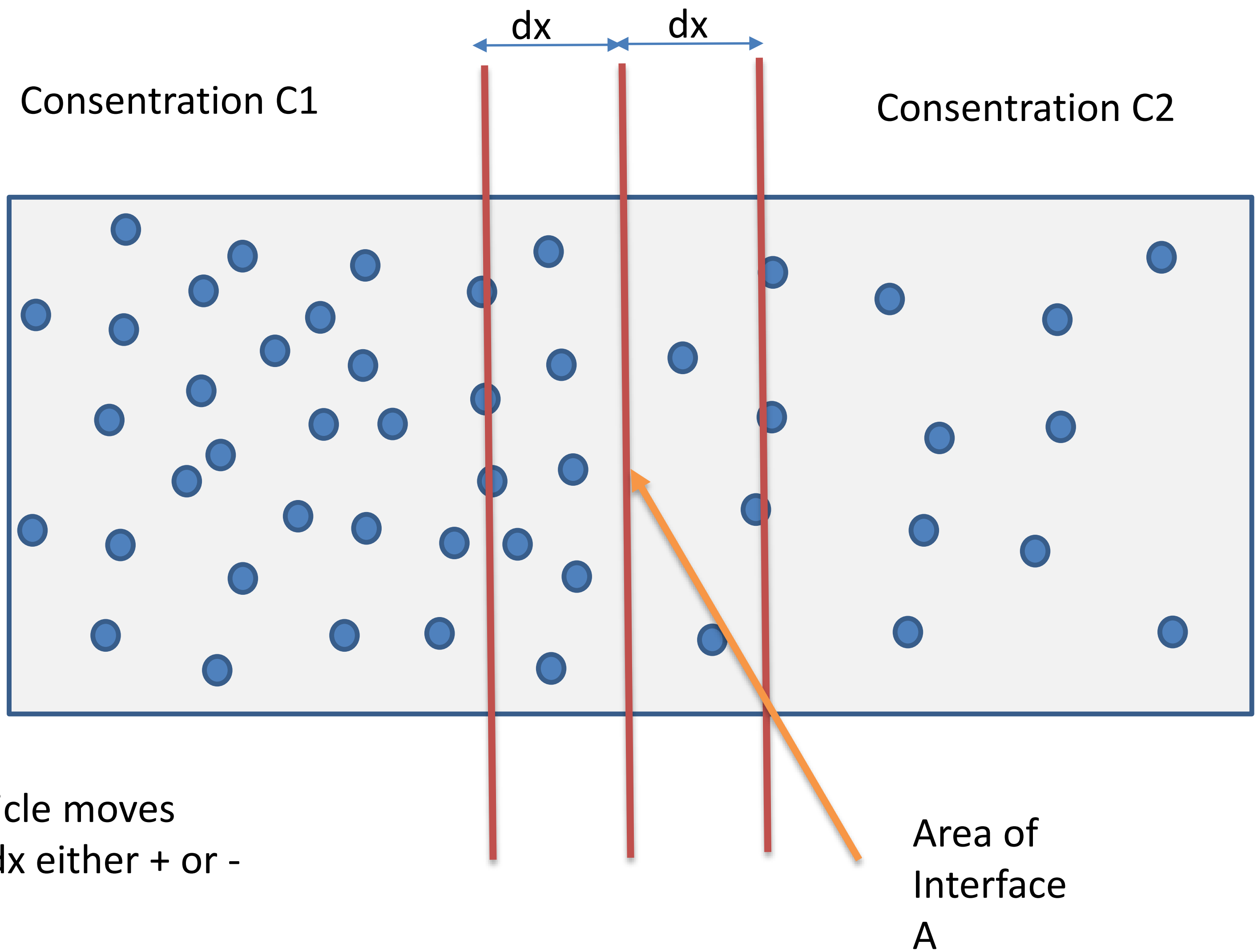
- $\underline{\sqrt{\langle d^2 \rangle} = \sqrt{N}}$

Random Walk as a Tree



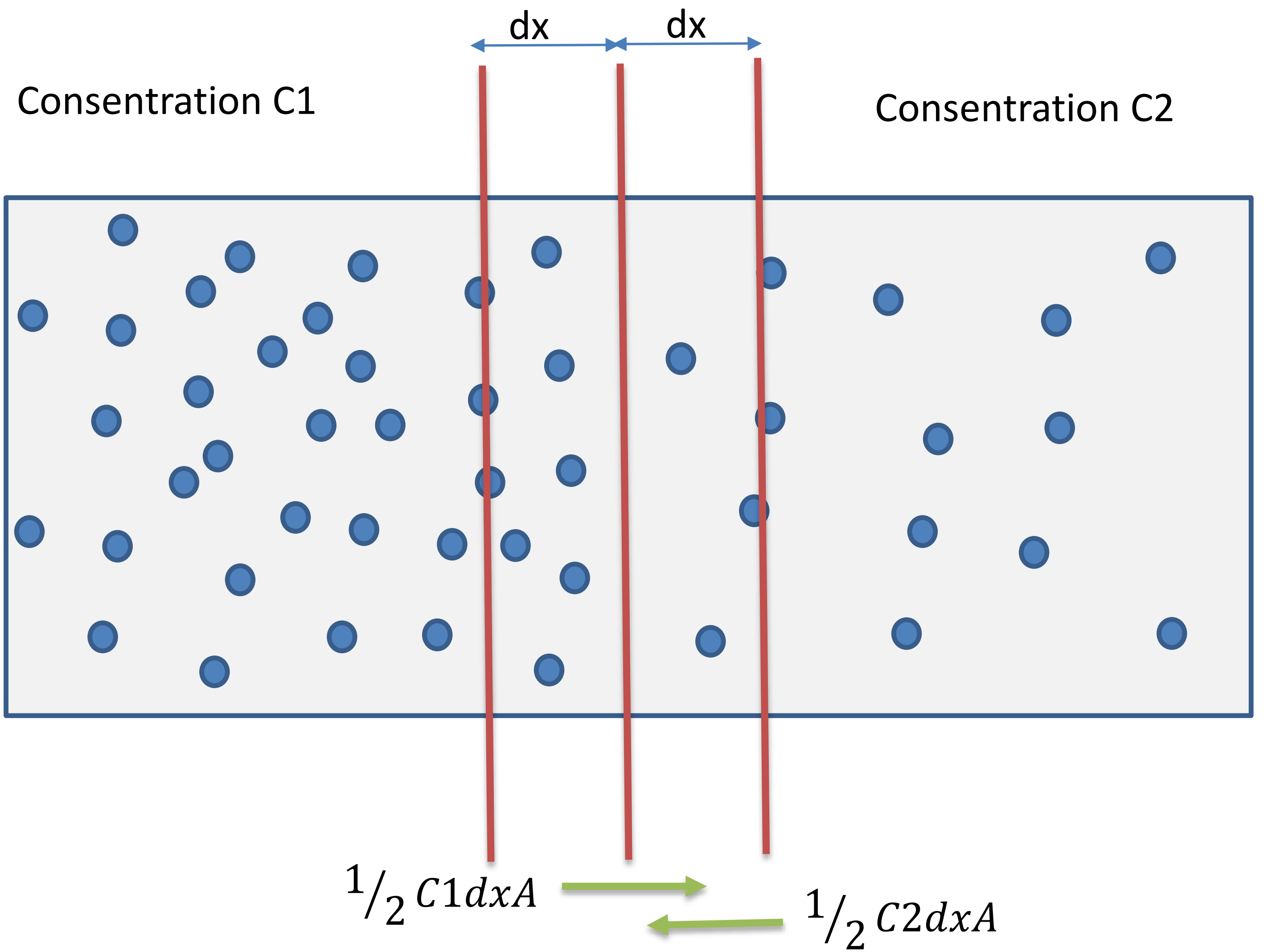
We can calculate the probability of a particle being at a given distance from the start point.

Molecule Diffusion



Each particle moves
distance dx either + or -

Molecule Diffusion



Diffusion – Fick's Law

$$q_x = \frac{0.5 \Delta X A (C1 - C2)}{\Delta t}$$

If $C(X)$ is continuous, then

$$C2 \approx C1 + \Delta X \; \partial C / \partial X$$

$$q_x \approx -\left[\frac{\Delta X^2}{2\Delta t}\right] A \frac{\partial C}{\partial X} = -DA \frac{\partial C}{\partial X}$$

D is diffusion coefficient $\quad D \sim (1/2) \, \Delta X^2 / \Delta t$

In Class Experiment

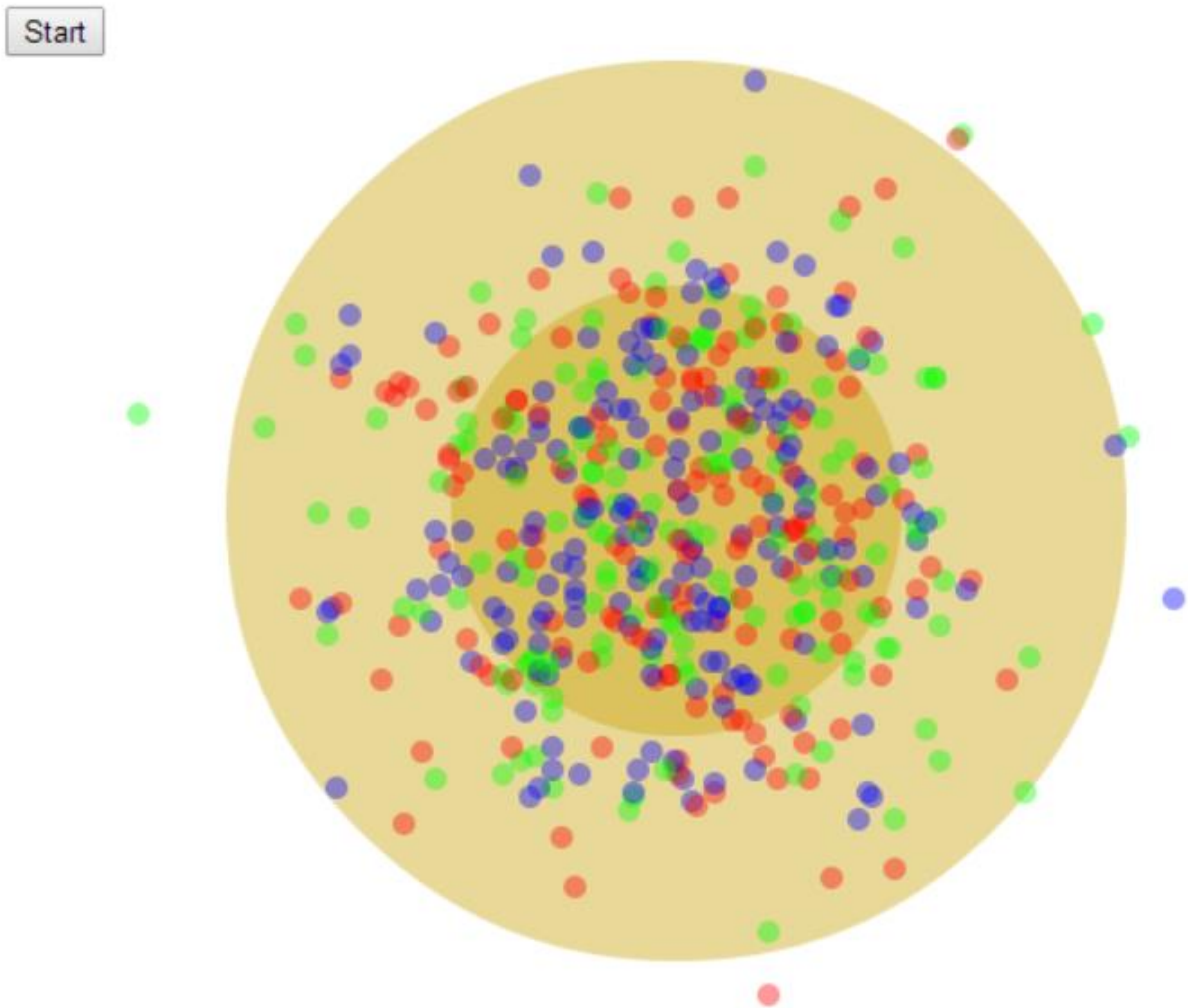
Make your code run in 2-D. Change the distance moved per step to be + or -1 in x and in y direction. Run with 200 particles.

Draw a fixed circle radius 100 and another with radius 200 with the center at the creation point for particles.

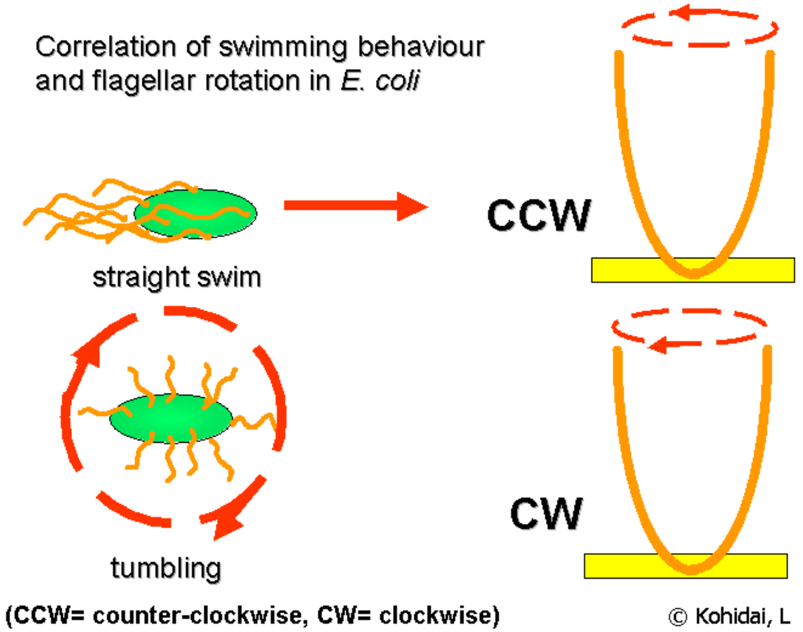
Compare the time for the first particle to cross the 100 circle with the time for one to cross the 200 circle by monitoring the distance of the particles from the center and writing the time step out to the console.

What should be the relationship between these times? Hand in a screen shot and the relationship you calculate.

$$\underline{\sqrt{\langle d^2 \rangle} = \sqrt{N}}$$

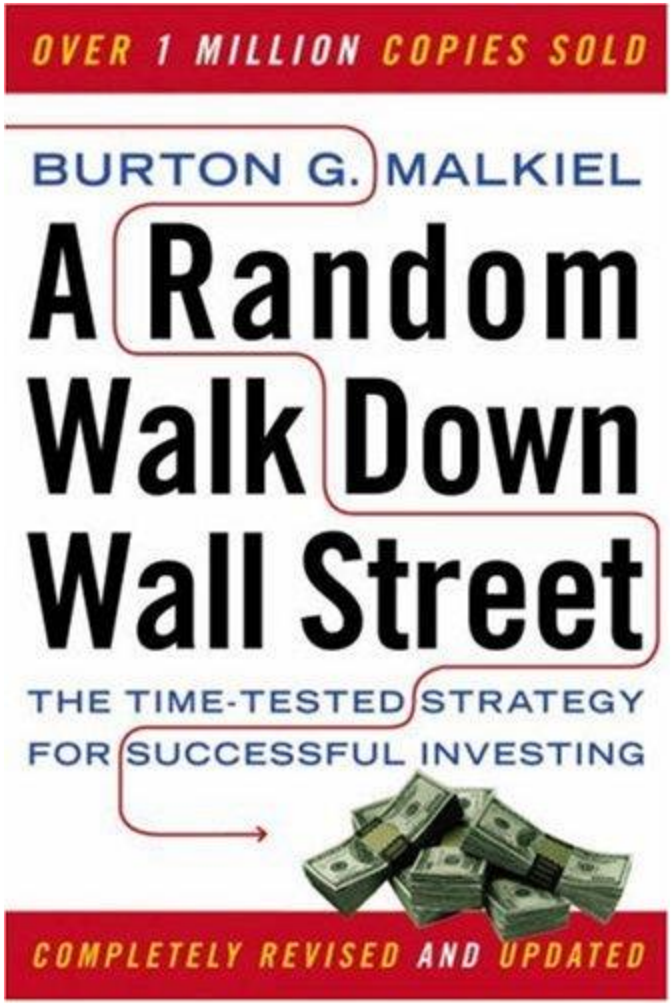

Start

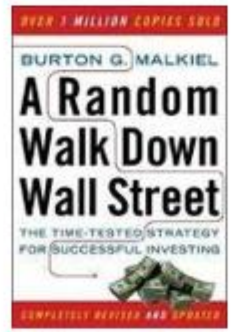
Random Walk of E.Coli



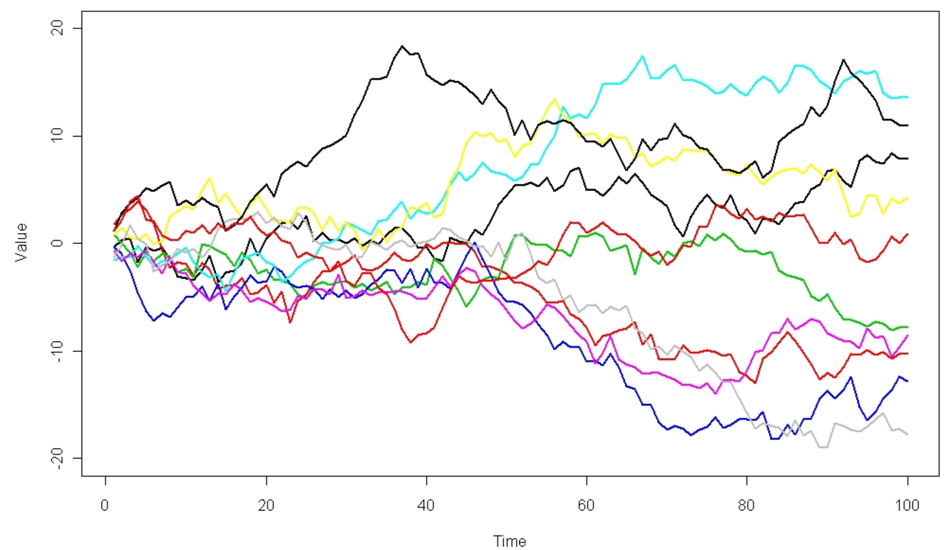
The movement looks like a random walk with relatively straight swims interrupted by random tumbles that reorient the bacterium. In the presence of a chemical gradient bacteria will chemotax, or direct their overall motion based on the gradient. http://en.wikipedia.org/wiki/Chemotaxis

Random Walk in Finance
(don’t bet on it)





vs

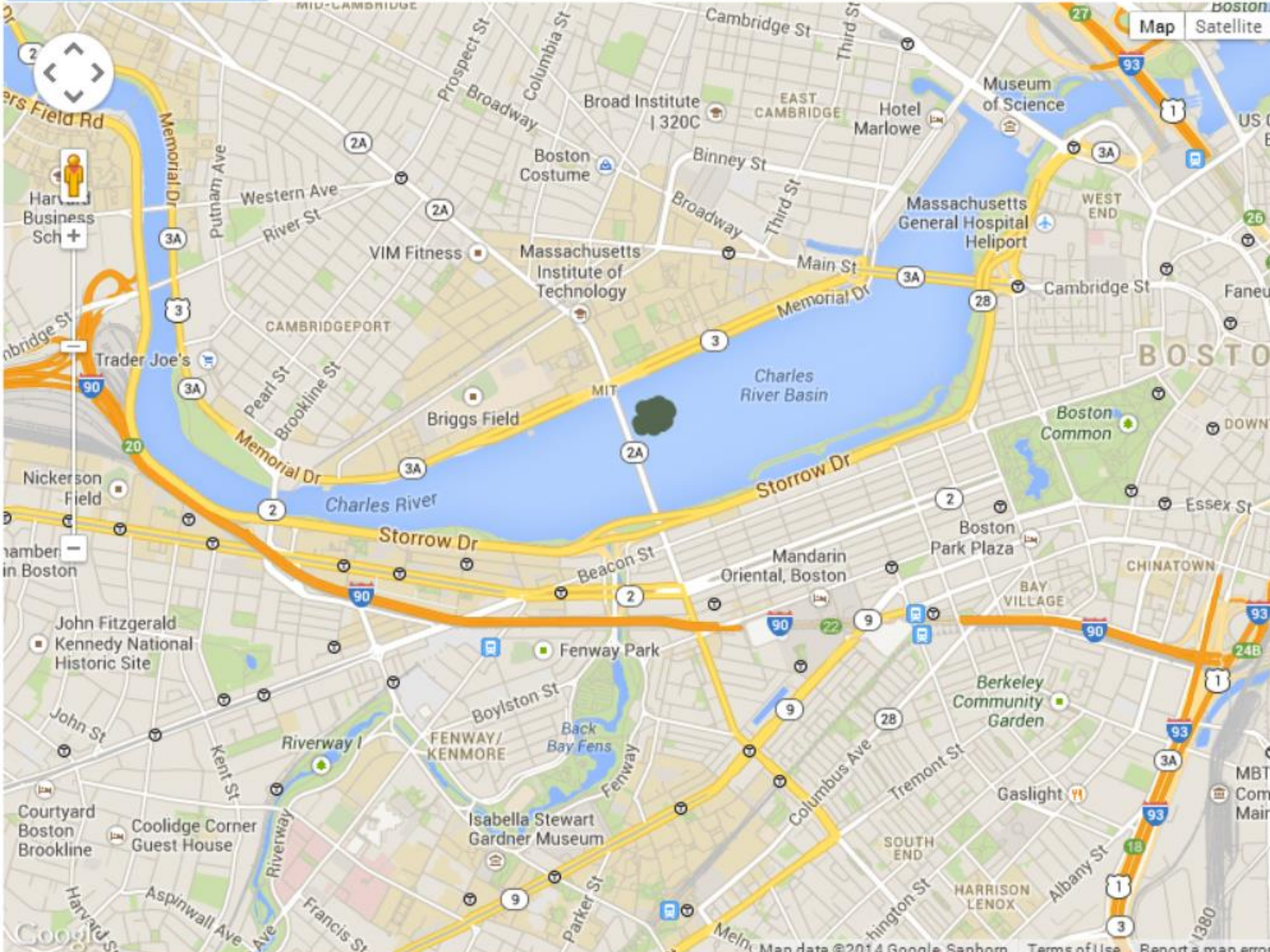


In Class Exercise 3

- Take the starter code for a small radioactive release near MIT. This is the same as the starter code for Problem Set 8. Start by making the random walk symmetric about the start point.
- Then add in the wind Vector(1,-1) - I'm assuming canvas cords so this is blowing towards the North East.

Apps Manage My Bookin... 1.00 Lectures Quiz 1 - Dropbox Grad Applications Abel PICKUP

Start Radiation Realease



Chicago Gas Release from Ship